Int. J. Heat Mass Transfer. Vol. 35, No. 1, pp. 121–129, 1992
Printed in Great Britain

0017–9310/92 $5.00 + 0.00

Group modeling of impacting spray dynamics

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(*Received* 4 *September* 1990 *and in final form* 17 *December* 1990)

Abstract—A 'Group' model has been developed for impacting sprays. This model uses droplet groups to represent the spray, employs the Lagrangian approach to trace the motion of groups, and considers the turbulent dispersion of droplets within each group. Each group has a dimension and grows during its motion. Droplet dispersion within a group is described by a probability density function. Application to impacting water sprays shows that this 'Group' model is very efficient and realistic. The computation time is about a factor of 20 times less than the corresponding calculation using the Stochastic Separated Flow (SSF) approach. The calculated results provide the important information of droplet diameter effects, droplet trajectories, impact velocities, and the impact angles of spray impaction dynamics.

1. INTRODUCTION

IMPACTING sprays are widely applied in many industrial processes, such as spray painting, spray coating, spray cooling and spray combustion, etc. The impact mechanism is very important in these processes. The impact angle and velocity of different size droplets affect strongly the uniformity of the deposition, the air bubble entrainment in the liquid film, and the heat transfer of sprays. In spray combustion, the impingement of fuel sprays on engine walls affects fuel evaporation, heat transfer, and coke deposition on the walls.

Although some fundamental research has been conducted for the drop–wall interaction, turbulent spray deposition modeling is still inadequate for general applications. The existing models of impacting sprays are either over-simplified or too tedious for implementation. For example, modeling of spray–wall impingement was studied by Naber and Reitz [1], and Reitz [2] by tracing single droplets in sprays. However, these computations require techniques involving tedious work and long computation times since a large number of computation droplets are needed for a realistic simulation; and therefore, are not easily acceptable for industrial use. Effective and accurate models are very much desired to provide predictions of processes and further improvement of products.

There are numerous difficulties in the development of efficient and realistic spray models. One of the major difficulties is the treatment of the turbulent interaction between gas phase and dispersed phase. Several approaches have been employed to model the turbulent droplet dispersion in sprays. A thorough review of these approaches was given by Faeth [3, 4]. Basically, two main methods have been employed to compute droplet dispersion in sprays. One is the Continuum approach, which considers the droplet dispersion as a diffusion process with an appropriate droplet diffusion coefficient. However, no general method exists for predicting droplet diffusion coefficients, and it is inconvenient to incorporate the model into the Lagrangian formulation of droplet tracing. The other is the Lagrangian approach where the droplet dispersion is modeled directly by considering the interaction between turbulent eddies and droplets in trajectory calculations.

Among the Lagrangian models, a rather advanced one is the Stochastic Separated Flow (SSF) model which has been widely accepted for the modeling of turbulent sprays, and also applied to the modeling of impacting sprays [1, 2]. The SSF model uses droplet clusters to represent a spray. Each cluster contains a large number of droplets which have the same properties. The trajectory of each cluster is calculated in Lagrangian coordinates as a series of interactions with pseudo-random turbulent eddies. In a sense, this approach is an exact simulation of dispersed-phase properties. The computation results showed good agreements with measurements [3, 5]. In the SSF approach, however, all the droplets within the cluster are assumed to be concentrated at one point during the flight, behaving like a single droplet at all times. A droplet cluster is essentially a computational droplet. It does not consider the turbulent dispersion of droplets which belong to the same cluster. Therefore, in order to obtain a statistically meaningful deposition distribution, a large number of characteristic droplet trajectories and, consequently, very long computation times are needed.

This paper presents a new 'Group' model for impacting sprays. The model uses droplet groups to represent a spray and takes into account the turbulent droplet dispersion within each group in the flow field. In the SSF model, however, all the droplets in a cluster are assumed to stay at the center of the cluster and a Dirac delta function is essentially used for the probability density function of droplet position within each cluster. The present 'Group' modeling considers that each group has a dimension which grows during the

NOMENCLATURE

C	droplet concentration
C_D	drag coefficient
C_μ	turbulence model constant
d	nozzle diameter
d_d	droplet diameter
h	heat transfer coefficient
k	turbulent kinetic energy
L_e	eddy characteristic size, $C_\mu^{3/4} k^{3/2}/\varepsilon$
$\dot{m}$	mass flux of gas
N	number of droplets in a group
r	radial coordinate
R_g	droplet group size
Re	Reynolds number, $\rho(\mathbf{u}-\mathbf{u}_d)d_d/\mu$
S	source term in governing equations for gas phase
t_e	eddy lifetime
t_R	droplet transit time
t_i	interaction time between droplet and eddy
$\mathbf{u}$	gas velocity vector
x	axial coordinate
$\bar{X}^2$, $\bar{Y}^2$	mean square distances
y	vertical distance.

Greek symbols

ε	dissipation rate of turbulence kinetic energy
μ	viscosity
μ_t	turbulent viscosity, $C_\mu \rho k^2/\varepsilon$
ρ	density
τ_r	droplet relaxation time
σ_ϕ	turbulent Prandtl/Schmidt numbers
ϕ	general variable.

Subscripts

d	droplet property
0	initial condition.

flight. This growth is due to the turbulent dispersion of droplets as the group travels in the Lagrangian coordinates. The droplet dispersion within a group is described by a probability density function which is a Gaussian distribution. Due to the growth of the group, only a small number of groups must be traced to provide statistically reasonable results.

The 'Group' approach closely simulates a real flow, because in a real flow the droplets within a group are dispersed by turbulence instead of staying at one point. This is consistent with the conceptual situation that if the droplets in a small region of a spray are dyed a red color, this dyed group of droplets will disperse in time within the spray. The present 'Group' approach essentially traces this dyed group of droplets in the spray. The concept of group modeling has already been presented by the authors [6]. In the present paper, the modeling is greatly refined with applications to impacting sprays also provided.

The concept of this 'Group' approach can be incorporated into any computational program which presently employs the Lagrangian approach to trace the dispersed phase. As will be discussed in later sections, the major assumption used in the 'Group' approach is that the turbulence should be isotropic in the field. This assumption is reasonable for most axisymmetric turbulent sprays which contain strong shear layers. In the shear layers of a spray, although the turbulent flow is not isotropic in the tangential direction, it is isotropic in the axial and radial directions. The present group model can be used directly on the calculations of two-dimensional axisymmetric sprays which are not dependent upon the tangential coordinate.

In the present study, droplet deposition on cold walls is also monitored to reveal the impaction dynamics of sprays. It is assumed that the droplet stays on the surface when contact occurs. This is one of three impingement models classified by Reitz [2]. Such an assumption is true for many practical situations, for example, the spray painting and the impacting spray heat transfer when wall temperature and mass flux are low. With the prediction of impacting spray dynamics, other subsequent phenomena such as heat transfer and evaporation processes on the surface can be modeled. For example, the droplets' impaction heat transfer results based upon the experimental study of ref. [7], and the analytical study of ref. [8] can be readily included to predict the spray cooling of high temperature materials.

The attention of this paper is to focus on the concept of the 'Group' approach for the modeling of sprays, and to take the impacting spray as an example of applications. It is not intended to elaborate on many details and possible further improvements of the model for other applications. This new modeling concept is expected to have more advantages when the geometrical scale is large, where its higher computational efficiency becomes indispensible.

2. BASIC EULERIAN/LAGRANGIAN FORMULATION

In the present study, a standard Eulerian approach is applied to the gas phase transport. The Lagrangian formulation is used to trace the motion of the center of each group. The modeling of the turbulent droplet dispersion in a group is a new approach.

2.1. *Gas phase*

The gas phase is represented by Eulerian conservation equations and the k–ε two-equation tur-

bulence model. The governing equations for a steady, axisymmetric turbulent flow are

$$\frac{\partial(\rho\bar{u}\phi)}{\partial x}+\frac{1}{r}\frac{\partial(r\rho\bar{v}\phi)}{\partial r}=\frac{\partial}{\partial x}\left(\frac{\mu_t}{\sigma_\phi}\frac{\partial\phi}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\mu_t}{\sigma_\phi}\frac{\partial\phi}{\partial r}\right)+S_\phi+S_{p\phi} \quad (1)$$

where the designation $\phi = 1$ yields the continuity equation. The symbol ϕ also represents the mean axial and radial velocity components $\bar{u}$ and $\bar{v}$, turbulent kinetic energy k, dissipation rate of turbulent kinetic energy ε, and total energy E. The turbulent Prandtl number is represented by σ_ϕ and S_ϕ is a general source term, while $S_{p\phi}$ represents the particular sources due to the presence of droplets. The expressions for S_ϕ and $S_{p\phi}$ pertaining to each variable are standard forms and given in ref. [9]. The constants used in the k–ε two-equation turbulence model are $C_\mu = 0.09$, $C_1 = 1.44$, and $C_2 = 1.92$.

2.2. *Dispersed phase*

The dispersed phase is represented by characteristic droplet groups, which are described with a Lagrangian view point. The center of a group is traced as a single computational droplet with its droplet motion equation, after neglecting the virtual mass, Bassett forces, and Magnus forces, given as

$$\frac{d\mathbf{u}_d}{dt}=\frac{(\mathbf{u}-\mathbf{u}_d)}{\tau_r} \quad (2)$$

where $\mathbf{u}$ and $\mathbf{u}_d$ are the local gas and droplet velocity vectors, and τ_r the droplet relaxation time.

The droplet relaxation time is calculated by [5]

$$\tau_r=\frac{4\rho_d d_d}{3\rho C_D\|\mathbf{u}-\mathbf{u}_d\|} \quad (3)$$

where C_D is the drag coefficient which is obtained from the following expressions:

$$C_D = 0.44 \quad \text{for} \quad Re > 1000 \quad (4)$$

$$C_D = 24(1+Re^{2/3}/6)/Re \quad \text{for} \quad Re < 1000. \quad (5)$$

The position of a computational droplet which represents a group center can be found by integrating

$$\frac{d\mathbf{x}_d}{dt}=\mathbf{u}_d. \quad (6)$$

3. TURBULENT EFFECTS

3.1. *Center of a group*

The turbulent effect on the trajectory of a computational droplet which represents the center of a group is considered in a fashion similar to the SSF method. The motion of the center of a droplet group is assumed to behave like a single droplet interacting with a succession of turbulent eddies [3, 4]. Properties within a particular eddy are uniform and determined from the gas phase analysis. The instantaneous velocity field that this computational droplet experiences consists of the mean and fluctuating gas velocities. The magnitude of the fluctuating velocity component is proportional to $(2k/3)^{1/2}$ (where k is the turbulent kinetic energy) by assuming that the turbulence is isotropic. The instantaneous gas flow velocities are randomly sampled when a computational droplet interacts with eddies. The interaction time between the droplet and the eddy is taken as the minimum of the eddy lifetime and the transit time required for the droplet to cross the eddy

$$t_i = \min(t_e, t_R) \quad (7)$$

where t_e is the eddy lifetime, and t_R the droplet transit time.

The eddy lifetime is calculated by [5]

$$t_e = L_e/(2k/3)^{1/2} \quad (8)$$

where L_e is the eddy characteristic size, and k the turbulent kinetic energy.

The droplet transit time is given by [5]

$$t_R = -\tau_r \ln\left(1.0-\frac{L_e}{\tau_r\|\mathbf{u}-\mathbf{u}_d\|}\right). \quad (9)$$

3.2. *Droplets in a group*

In an actual turbulent two-phase flow, the droplets belonging to a group are dispersed by turbulence rather than staying concentrated at one point during the flight. Conceptually, a group of droplets in a spray is viewed as if the droplets in this group are dyed a red color. These red droplets disperse in the spray; however, we are only tracing these red drops with the assumption that there are no collisions and coalescence. According to the present model, all the droplets in a group move together in an absolute coordinate and the droplet dispersion is a relative motion to the center of the group. Therefore, the droplet group grows from its center, and the dispersion of droplets is observed from the center of the group.

This paper presents a new model which will treat the turbulent dispersion of droplets within a group under the following assumptions:

(1) Drop-to-drop collisions are neglected.

(2) The turbulent flow is 'locally' isotropic and homogeneous.

(3) The dispersion of droplets in a group is assumed to originate from a point source located at its center when this group leaves the nozzle.

Although the center of a group follows the trajectory of a single droplet, the relative droplet position within a group in respect to the group center at any time is random because of the turbulent gas velocity field. Therefore, the movement of droplets relative to the center of the group is almost like a random walk. Theoretical considerations and experimental data suggest that the probability of a droplet transport by turbulent diffusion is similar to Brownian diffusion [10]. This kind of droplet dispersion can be closely

represented by a normal Gaussian distribution. The typical probability density function for this distribution can be given for a two-dimensional field as

$$f(x,y,t)=\frac{N}{\sqrt{(2\pi\bar{X}^2)}\sqrt{(2\pi\bar{Y}^2)}}\exp\left(-\frac{x^2}{2\bar{X}^2}-\frac{y^2}{2\bar{Y}^2}\right) \tag{10}$$

where N is the total droplet number in this group, and $\bar{X}^2$ and $\bar{Y}^2$ are the overall mean square distances in the x- and y-directions. The $\bar{X}^2$ and $\bar{Y}^2$ are functions of time and are affected by the droplet size and the gas flow field. All the measurements of x and y are made from the center of the group. The total droplet group size is estimated to be $6\sqrt{(\bar{X}^2)}$ and $6\sqrt{(\bar{Y}^2)}$ based on the probability theory.

It is interesting to note that equation (10) is also a solution to a diffusion equation. One can prove that this distribution preserves its Gaussian form when droplets encounter different eddies in the flight (see Appendix). The overall mean square distances of equation (10) can be calculated from

$$\bar{X}^2=\delta_{x1}^2+\delta_{x2}^2+\cdots+\delta_{xi}^2+\cdots \tag{11}$$

$$\bar{Y}^2=\delta_{y1}^2+\delta_{y2}^2+\cdots+\delta_{yi}^2+\cdots \tag{12}$$

where δ_{xi}^2 and δ_{yi}^2 are the mean square displacements in each time interval δt_i which can be calculated from the turbulent diffusion theory [11]

$$\delta_{xi}^2=u_d'^2\delta t_i^2;\quad \delta_{yi}^2=v_d'^2\delta t_i^2. \tag{13}$$

In equation (13), u_d' and v_d' are droplet fluctuating velocities. They can be calculated by the following equation, which is obtained by subtracting the mean flow equation (2) with the instantaneous equation of motion for a droplet:

$$\frac{d\mathbf{u}_d'}{dt}=\frac{1}{\tau_r}(\mathbf{u}'-\mathbf{u}_d') \tag{14}$$

where $\mathbf{u}'$ and $\mathbf{u}_d'$ are the gas and droplet fluctuating velocity vectors, and τ_r the droplet relaxation time.

4. NUMERICAL ANALYSIS

To illustrate this 'Group' approach, the gas phase elliptic partial differential equations together with the boundary conditions are solved using an existing finite difference computer program, TEACH-code [12], which is widely applied for the computation of turbulent flows. The droplet–gas coupling is incorporated in the numerical procedure. In a highly dilute two-phase flow, using a one-way coupling, which neglects the droplet effects on the gas phase, will eventually give the same results as that using a two-way coupling, which uses special source terms to consider the presence of droplets. The ordinary differential equations governing the behavior of each computational droplet are solved by a second-order accurate implicit method for each computational droplet. For the droplet dispersion within a group, the distribution function is calculated in every time step. The integration of equation (10) gives the droplet numbers in the computational cells covered by a group

$$N_{cell}=N\int_{x_1}^{x_2}\frac{1}{\sqrt{(2\pi\bar{X}^2)}}\exp\left(-\frac{x^2}{2\bar{X}^2}\right)dx\times\int_{y_1}^{y_2}\frac{1}{\sqrt{(2\pi\bar{Y}^2)}}\exp\left(-\frac{y^2}{2\bar{Y}^2}\right)dy \tag{15}$$

$$N_{cell}=\frac{N}{4}\left[\mathrm{erf}\left(\frac{x_2}{\sqrt{(2\bar{X}^2)}}\right)-\mathrm{erf}\left(\frac{x_1}{\sqrt{(2\bar{X}^2)}}\right)\right]\times\left[\mathrm{erf}\left(\frac{y_2}{\sqrt{(2\bar{Y}^2)}}\right)-\mathrm{erf}\left(\frac{y_1}{\sqrt{(2\bar{Y}^2)}}\right)\right] \tag{16}$$

where x_1, x_2, y_1, and y_2 are the boundary positions of the computational cell.

5. RESULTS AND DISCUSSION

5.1. *General nature*

Hinze [11] developed an analytical expression for the turbulent dispersion of 'marked' fluid droplets. The droplets were introduced into a uniform turbulent flow field at a constant rate from a point source. The concentration distribution $C(x, r)$ at position (x, r) for homogeneous and isotropic turbulence with long diffusion times is as follows [11]:

$$C(x,r)=\frac{S_v}{4\pi\mu_t|x|}\exp\left(-\frac{Ur^2}{4\mu_t|x|}\right) \tag{17}$$

where U is the flow velocity, μ_t the turbulent diffusivity, and S_v the volumetric source strength.

Equation (17) is essentially of a Gaussian form. As shown in Fig. 1, the distribution follows Gaussian form after normalization, where $\bar{C}=C/(S_v/4\pi\mu_t|x|)$ and $\bar{r}=r/(\sqrt{(2\mu_t|x|/U)})$. This is consistent with the present assumption of Gaussian dispersion of droplets within each group.

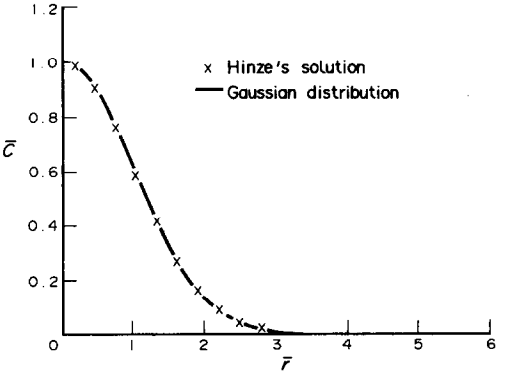

FIG. 1. Normalized radial particle concentration distributions.

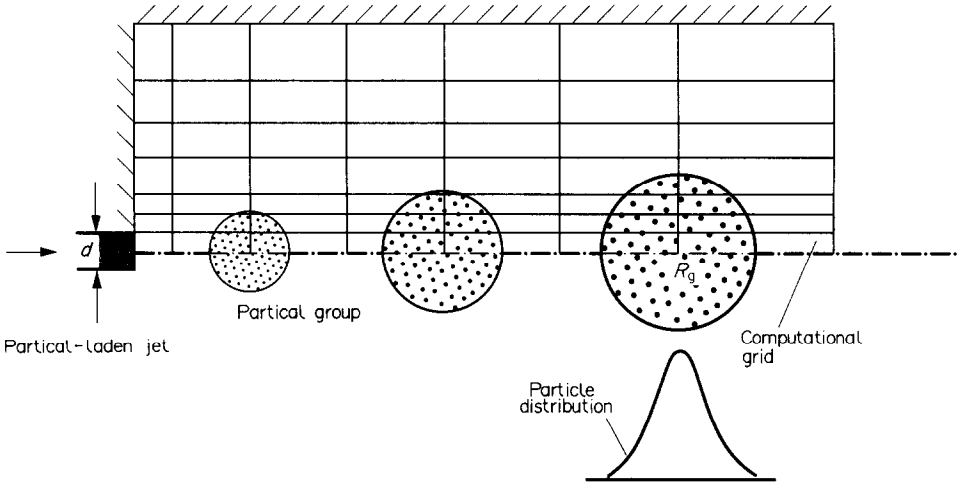

FIG. 2. Schematic of the computational flow field.

5.2. *Data comparisons on particle laden jets*

In order to validate the new approach, the 'Group' modeling was applied to the analysis of a particle-laden jet containing fly ash particles. The preliminary results of this study were compared with the results of the SSF method and the experimental data.

The experimental results were reported by Yuu *et al.* [13]. Their experimental set-up contained a jet nozzle which could provide uniform exit properties. This flow uniformity allowed a plug flow assumption to be imposed as an initial condition for the numerical calculations. The nozzle geometry and the properties of the two-phase particulate flow were taken from ref. [13] and included in the computations to enable comparisons with the experimental results. The nozzle diameter is 8 mm. The particles are nearly monodisperse having a mass mean diameter of 20 μm. The flow is dilute with particle mass loadings of 0.1–0.4%. These loadings are sufficiently small, so that the particles have a negligible effect on the mean and turbulent gas phase properties. The grid of the flow field is axial system with 32×29 meshes. The computational flow field is shown schematically in Fig. 2.

The distributions of particle concentrations along the center line of the jet are shown in Fig. 3 for the present method, the SSF approach, and the experimental results. The SSF results are presented for two different particle cluster numbers (20 and 500). For the smaller particle cluster number, the particle concentration distribution is not smooth due to statistically insufficient data. In order to obtain a reasonable agreement with the experimental results, 500 particle clusters were required as shown in Fig. 3. Unlike the results of the SSF method, the results of the present approach are in sound agreement with the experimental results with only 15 groups. At $x/d > 15$, slightly larger deviation is observed.

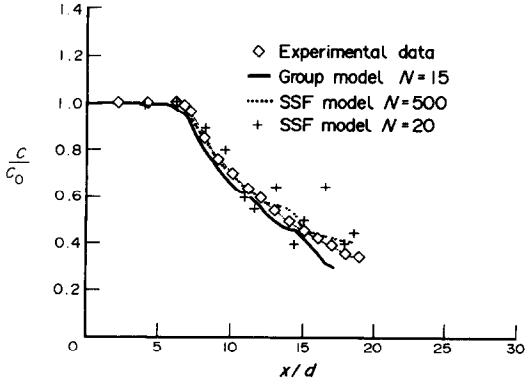

FIG. 3. Axial particle concentration distributions.

The difficulty with the predictions downstream can be attributed to the fact that the assumption of locally homogeneous turbulence within the group becomes incorrect when a group grows to a very large size. This problem can be solved with group splitting where the group number can be less than 15 when leaving the nozzle; however, the group number increases after splitting occurs downstream. The concept of group splitting is straightforward but its implication contains some arbitrary criteria which are case dependent. Since the present paper is emphasizing the group concept and its application to impacting sprays where the droplet dispersion is not very severe, the details of group splitting will be discussed in a different paper in the future.

5.3. *Impacting sprays*

Impacting sprays were selected as an example to illustrate the capability of the 'Group' model. This selection was chosen because impacting sprays contain droplets of various sizes under severe flow conditions, and there are not many detailed investigations reported in the literature despite its wide applications. Impingement of axisymmetric water sprays from a 9.5 mm diameter nozzle normally on a flat plate was studied using a computer program employing the 'Group' approach. The distance between the plate and the exit of the injector is 30.5 cm. The gas phase is assumed to have a plug flow velocity profile at the nozzle

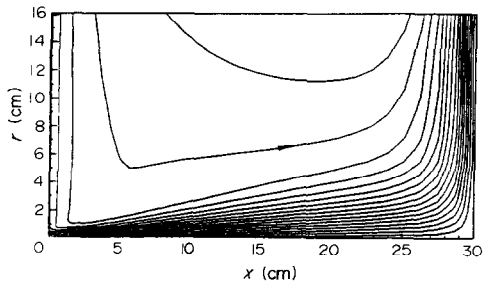

FIG. 4. Gas flow streamlines.

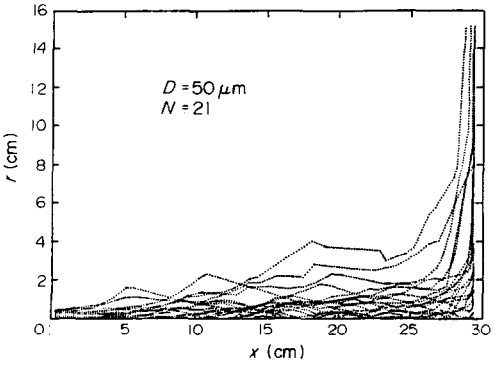

FIG. 6. The trajectories of group centers, $D = 50$ μm.

exit with 3 m s^{-1} velocity. The droplets are assumed to be monodisperse. The effect of droplet size on the impingement characteristics was investigated for the droplet diameters 20, 50, 100, and 150 μm, respectively. The spray flow is considered to be dilute with an initial liquid volume flow rate fraction of 10^{-5}.

The gas phase mean flow field is shown in Fig. 4. The mean flow streamlines demonstrated three flow regimes which were defined in ref. [14]. The free jet regime is upstream of jet impingement. The strong interaction of the jet with the impingement surface produces a change in flow direction in the impingement regime. The wall jet regime consists of radial flow along the surface beyond the point at which the strong impingement to the wall occurs.

Figures 5–7 show the trajectories of droplet group centers for the droplet diameters 150, 50, and 20 μm, respectively. The momenta of droplets increase with their diameters. Larger diameter droplets, such as those with a 150 μm diameter, tend to continue in their initial direction with less influence by the gas flow field in comparison to smaller diameter droplets. The resulting trajectories stay closer to the symmetry axis with little droplet dispersion as shown in Fig. 5. The smoothness of the trajectories implies that the flow turbulence also has a minimum influence for these conditions.

As the droplet diameter decreases, the 50 μm diameter droplets follow the mean gas flow more closely, as indicated in Fig. 6. This is evidenced by the nearly parallel direction of the trajectories close to the plate surface. The trajectories are also affected by the flow turbulence. The wider spread of 50 μm diameter droplets compared to that of 150 μm diameter droplets shows the results of the interaction between the fine droplets and the turbulent eddies. Figure 7 displays the trajectories of 20 μm diameter droplets. Because of their small momenta, these droplets follow the gas flow field very closely and the spread of the trajectories is similar to that of the gas flow streamlines. The irregularity of trajectories and the significant dispersion of droplets reveal that the interaction between the turbulent eddies and the droplets is the most effective for the 20 μm diameter droplets. In the calculations of trajectories, when droplets intersect the symmetric axis, they are treated as 'reflection' according to the mass conservation. This can be observed from Figs. 6 and 7.

The radial distributions of the droplet deposition flux on the wall are shown in Fig. 8 for the droplet diameters 20, 50, 100, and 150 μm, respectively. This droplet deposition flux is defined as the volume of total droplet deposition per unit wall area per unit time. A deposition is assumed to occur when a droplet approaches the wall surface within a distance of 0.01 cm. For all the cases compared in Fig. 8, the droplet volume flow rate at the nozzle and the droplet group numbers (21 groups) in computations are all the same,

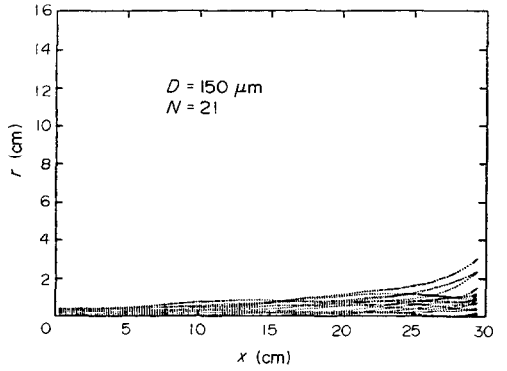

FIG. 5. The trajectories of group centers, $D = 150$ μm.

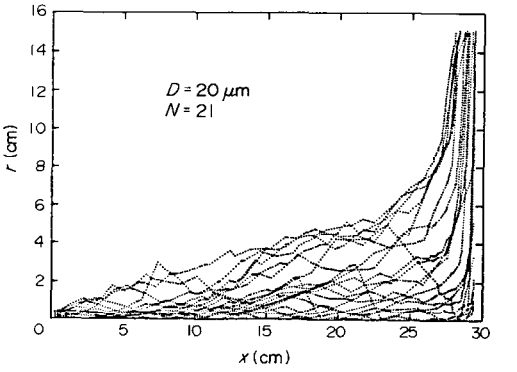

FIG. 7. The trajectories of group centers, $D = 20$ μm.

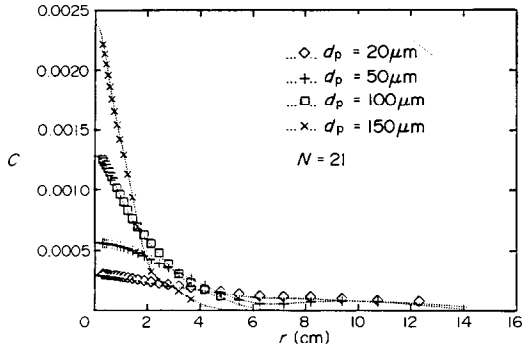

FIG. 8. Radial distributions of droplet deposition flux.

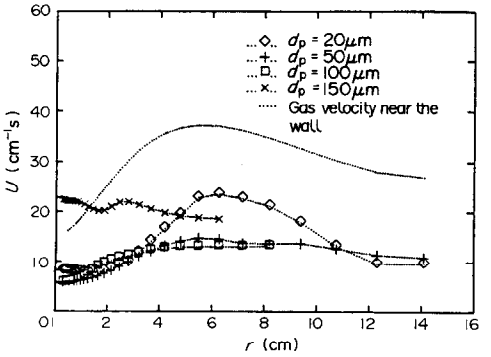

FIG. 10. Impact velocity distributions on the wall.

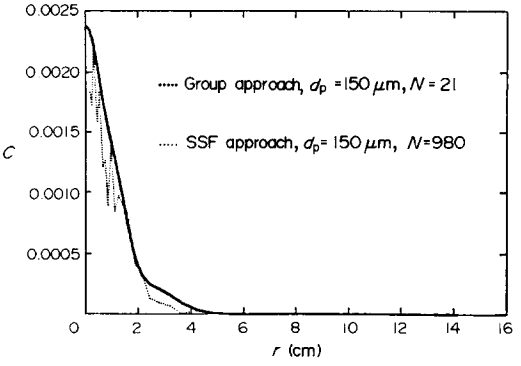

FIG. 9. Radial distributions of droplet deposition flux.

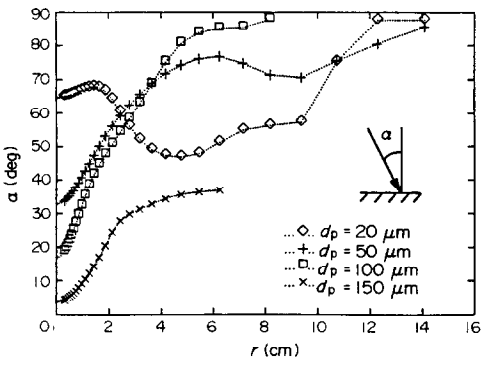

FIG. 11. Impact angle distributions on the wall.

but the droplet numbers contained in each group vary with droplet size. The distribution of droplet deposition flux for the different diameters is consistent with the implication of the trajectories in Figs. 5–7. The smaller diameter droplets spread wider and impinge over all the wall surface. With increasing droplet diameters, the deposition flux distribution increases at the central locations, and the radial extent of impingements decreases. The maximum deposition flux at the center is observed for 150 μm diameter droplets.

In the calculations, it appears that above $N = 21$ (where N is the group number), the distributions do not change significantly and further increases in the group number does not provide additional improvement. A comparison of the group approach with the SSF approach was also made in the present study. Figure 9 shows that the SSF approach with 980 clusters yields results similar to the present group calculations; however, the SSF distribution is not as smooth. Therefore, more realistic results can be obtained with only 21 groups in the 'Group' approach, as compared to 980 clusters of the SSF method for impacting sprays.

Figure 10 displays the radial distributions of overall droplet impingement velocities for the four different droplet diameters together with the overall gas velocity distribution near the wall. The distribution for 20 μm droplets shows a shape similar to that of the gas velocity near the wall. For 50 and 100 μm droplets, the velocity increases with radius and then remains almost constant in the wall jet regime (after about a 6 cm distance from the symmetry axis). The impact velocity distribution for 150 μm diameter droplets does not seem to be influenced by the gas velocity distribution.

The radial distributions of locally averaged impact angles for the four different droplet diameters are shown in Fig. 11. The impact angle, α, is defined as the angle between the drop trajectory and the direction normal to the wall. A zero degree impact angle means the impact is normal to the wall. For 150, 100, and 20 μm droplets, the impact angle increases with distance from the symmetry axis due to the bending of the gas flow along the wall surface. In the area near the symmetry axis, the impact angle increases as the droplet diameter decreases because the smaller droplets follow the gas flow more closely. The 150 μm diameter droplets have the smallest impact angles due to their high momentum. The distributions of 20 and 50 μm droplets are different from the other distributions. Their impact angle is smaller than expected in the area between 2 and 10 cm wall radii. It is noted that much of these small diameter droplets do not impact the surface of the plate and go out following the gas flow. For the droplets impacting the surface, they may

impact at small angles due to the effect of gas flow turbulence. The average impact angles obtained from these few impacts, therefore, gives lower impacting angles within the 2–10 cm wall radius range as shown in Fig. 11.

5.4. *Computational effort*

In the calculations of impacting sprays, a 36×35 grid was used, while for particle laden jets, a 32×29 grid was taken; negligible difference was observed when the number of grids was increased to 40×40. All calculations were performed on a Sun 386i/250 computer (5 MIPS, 25 MHz). In the calculations of impacting sprays, for the liquid phase calculation of 150 μm diameter droplets, the SSF approach took about 900 s with 980 groups, while the 'Group' approach only took about 46 s for similar results. The reduction in computation time for the impacting spray was about a factor of 20. In the calculations of particle laden jets, for the dispersed phase calculation, the computational time using the SSF approach with 500 groups was 509 s, while the present study only took 53 s for a similar agreement with the experimental data. The reduction in computation time for the particle laden jet was about a factor of 10. This major reduction in the computation time in the present approach is because of the significant decrease in the number of droplet groups required for realistic predictions. As expected, the reduction of computation time is more significant when the geometrical scale is large.

6. CONCLUSIONS

In this study, a new, efficient 'Group' model was presented by incorporating the turbulent dispersion of the droplets within a group while tracing the trajectory of its center stochastically. This approach can be applied to general computational programs which employ the Lagrangian method to trace the dispersed phase. The major advantages of the 'Group' model are as follows:

(a) Much fewer droplet group numbers are required to obtain reasonable statistical results than with the classical SSF approach. As a result, a significant reduction in computational time is achieved.

(b) A probability density function is used to describe the droplet distribution within the group, giving a more realistic description of the two-phase flow field.

The computational results using the 'Group' model on impacting sprays show that the droplet size has a strong effect on droplet dispersion and drop–wall impaction dynamics. Smaller droplets are affected and dispersed more by the turbulent gas field. The information on spray trajectories, droplet deposition flux, impact velocity, and impact angle, etc. can be obtained easily. This understanding allows for advances on various impacting spray applications.

Although the present paper emphasizes impacting sprays, the developed 'Group' approach is useful for other turbulent spray applications.

Acknowledgement—Partial support of DOE grant DE-FG22-88PC88932 is gratefully appreciated by the authors.

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APPENDIX

It has been proven that equation (10) preserves its form during the process. This proof is given as follows.

There is a well-known central limit theorem of the probability theory [15]. The theory states that if X_1 and X_2 are independent stochastic variables, having Gaussian distributions $N(0, \sigma_1^2)$ and $N(0, \sigma_2^2)$, respectively (where their averages are zero and their variances are σ_1^2 and σ_2^2, respectively), then their sum $X = X_1 + X_2$ is also Gaussian $N(0, \sigma_1^2 + \sigma_2^2)$. The average and variance of X are the sums of the averages and the variances of X_1 and X_2. Mathematically, this theory claims that if $f_{X_1}(x_1)$ and $f_{X_2}(x_2)$ are Gaussian probability densities of X_1 and X_2, respectively, the probability density of the sum, $f_X(x)$, will be

$$f_X(x) = \int f_{X_1}(x_1) f_{X_2}(x - x_1)\, \mathrm{d}x_1 \tag{A1}$$

$$= \frac{1}{\sqrt{(2\pi(\sigma_1^2 + \sigma_2^2))}} \exp\left(-\frac{x^2}{2(\sigma_1^2 + \sigma_2^2)}\right) \tag{A2}$$

which is also Gaussian distribution.

For the droplet dispersions within a group, the probability

function of droplet positions is essentially a solution of a diffusion equation. It is assumed that the diffusion coefficient is constant during each time interval δt, but varies with different time intervals when droplets encounter different eddies in the flight. The solution to the diffusion equation with an initial condition of a Dirac function will develop into a Gaussian distribution during a time interval δt_1 from time t_0 to t_1. Now, consider a time interval δt_2 from time t_1 to t_2. The final probability density will be

$$f_X(x,t_2) = N\int\left(\frac{\exp\left(-\dfrac{x_1^2}{2(2D_1\delta t_1)}\right)}{\sqrt{(2\pi(2D_1\delta t_1))}}\right) \times\left(\frac{\exp\left(-\dfrac{(x-x_1^2)}{2(2D_2\delta t_2)}\right)}{\sqrt{(2\pi(2D_2\delta t_2))}}\right)\mathrm{d}x_1 \quad \text{(A3)}$$

where D_1 and D_2 are diffusion coefficients. The first term in the integral is the initial condition of the diffusion equation from time t_1 to t_2, which is a Gaussian distribution after experiencing δt_1 time of diffusion. The second term in the integral is the Green's function of the diffusion equation. These two terms are similar and can be rewritten as

$$f_X(x,t_2) = N\int f_{X_1}(x_1)f_{X_2}(x-x_1)\,\mathrm{d}x_1. \quad \text{(A4)}$$

Equations (A4) and (A1) have the same form. Since the successive droplet displacements are uncorrelated, equation (A2) can be applied to the present situation. This means the final distribution is Gaussian and can generally be written as

$$f_X(x,t) = \frac{N}{\sqrt{(2\pi\bar{X}^2)}}\exp\left(-\frac{x^2}{2\bar{X}^2}\right) \quad \text{(A5)}$$

where

$$\bar{X}^2 = \delta_{x1}^2+\delta_{x2}^2+\cdots+\delta_{xi}^2+\cdots = 2D_1\delta t_1+2D_2\delta t_2+\cdots+2D_i\delta t_i+\cdots.$$

MODELISATION DE GROUPE DE LA DYNAMIQUE D'UN AEROSOL IMPACTANT

Résumé—On développe un modèle de "groupe" pour des aérosols impactants. Ce modèle utilise des groupes de gouttelettes pour représenter l'aérosol, il emploie l'approche Lagrangienne pour tracer le mouvement des groupes et considère la dispersion turbulente de gouttelettes dans chaque groupe. Celui-ci a une dimension et une croissance pendant son mouvement. La dispersion de la gouttelette dans un groupe est décrite par une fonction de distribution de densité de probabilité. L'application des aérosols d'eau impactants montre que ce modèle de groupe est très efficace et réaliste. Le temps de calcul est environ 20 fois plus faible que pour les calculs correspondants qui utilisent l'approche de l'écoulement séparé stochastique (SSF). Le calcul fournit une information importante sur les effets du diamètre de gouttelette, les trajectoires de gouttelette, les vitesses d'impaction et la dynamique d'impaction tenant compte des angles d'impact.

EIN GRUPPENMODELL FÜR DAS DYNAMISCHE VERHALTEN AUFTREFFENDER SPRÜHNEBEL

Zusammenfassung—Für auftreffende Sprühnebel wird ein "Gruppenmodell" entwickelt. Zur Darstellung des Sprühnebels benutzt es Tröpfchengruppen, deren Bewegung durch eine Betrachtung nach Lagrange beschrieben wird. Es berücksichtigt die turbulente Dispersion der Tröpfchen innerhalb jeder Gruppe. Jede Gruppe hat eine Größe und wächst während ihrer Bewegung. Die Tröpfchenverteilung innerhalb der Gruppe wird mit einer Verteilungsfunktion für die Wahrscheinlichkeitsdichte beschrieben. In der Anwendung auf auftreffende Wassersprühnebel erweist sich das "Gruppenmodell" als sehr leistungsfähig und realistisch. Die Rechenzeit ist um ungefähr einen Faktor 20 geringer als bei einer entsprechenden Berechnung mit der stochastischen Näherung für abgelöste Strömungen. Die berechneten Ergebnisse liefern wichtige Angaben über die Dynamik auftreffender Sprühnebel: Einfluß des Tröpfchendurchmessers, Trajektorien der Tröpfchen, Auftreffgeschwindigkeit und Auftreffwinkel.

ГРУППОВОЕ МОДЕЛИРОВАНИЕ ДИНАМИКИ СОУДАРЯЮЩИХСЯ СТРУЙ

Аннотация—Разработана "групповая" модель соударяющихся струй. В модели струя представляется группами капель, а для прослеживания за их движением используется подход Лагранжа, а также учитывается турбулентная дисперсия капель в каждой группе. Каждая группа имеет размерность и увеличивается по мере движения. Дисперсия капель в группе описывается функцией плотности вероятностей. Применение модели к соударяющимся струям воды показывает, что она является эффективной и реалистичной. Время вычисления примерно в 20 раз меньше, чем в случае использования стохастического метода разделенных потоков. Результаты расчетов дают важную информацию об эффектах диаметров капель, траекториях их движения, а также скоростях и углах соударения.